Public debt and safety trap in open economies

Valentin Marchal

May 2020

Abstract

Over the past decade, interest rates on safe assets have reached the Zero Lower Bound in Europe as in the USA. In the same time, the return on risky assets has remained quiet stable, leading to an increase in risk premium. There is hence evidences for a shortage of safe storage technology that has been defined as a "safety trap" in Caballero and Farhi (2018).

In this master thesis, we extend their model to an open economy with two countries: Home and Foreign. Home is assumed to produce more private safe assets than Foreign and to be able to issue safe public debt. Domestic government backs its debt on future labour or capital revenues. Taxes are raised first without and then with distortion costs. Our main finding is that allowing foreign agents to save in domestic safe assets is costly for Home. This cost is reduced to zero if, and only if, the world ends up outside the safety trap, eventually through public debt emission backed, without distortion costs, on capital incomes.

I would like to specifically thank my advisor Nicolas Coeurdacier for the time he spent attentively following my work. All along the research process his comments and advices have been precious in guiding me without ever being directive. I would also like to thank Xavier Ragot for introducing me to the safe assets literature when I solely had a broad interest for public debt.

1 Introduction

Since the 2008 financial crisis, the ECB has decreased its interest rate on deposit facilities from more than 3% to zero in 2012. It has not rebounded ever since and has decreased into negative territories. These low policy nominal interest rates could be the sign of a liquidity trap. This concept considers on one side safe and risky assets (taken as substitutable) and on the other side money. It assumes that agents have a speculative motive to save in money. After an increase in r, investors that have been saving in money would indeed be able to easily mobilize their savings for new investments.

This reasoning holds as long as we can abstract from risk preferences. But, while safe interest rates have decreased, expected rate of return on risky assets has remained quit stable. Geis et al. (2018) approximates it by studying the expected return on European listed equity. This latter has been contained between 7% and 10% over the period 2008-2018. It follows an increase in risk-premium. Thus, we cannot think current situation as a liquidity trap. At the ZLB, money has the same return and safety than other safe assets. Agents may therefore hold money, not as a result of speculative motives, but rather as a consequence of their risk aversion.

These considerations led Caballero and Farhi (2018) to introduce the concept of Safety Trap. It considers a time continuous OLG model. The economy is populated by infinitely risk-averse agents, the Knightians, and risk-neutral agents, the Neutrals. Potential output is given by Lucas trees and endowments of newborns. Aggregate risk is modelized by a single possible shock that can be either positive or negative. Neutrals own the risky Lucas trees and leverage them to sell safe assets to Knightians. Leverage possibilities are constrained by an exogenous securitization constraint: only a share ρ of future dividends can be pledged. Agents consume when they die. This stylized fact emphasises the importance of storage technologies.

Money is assumed to exist, but in zero (net or gross) supply. It implies that nominal interest rate on safe assets cannot be negative. In the benchmark model, absolute price rigidity for goods is assumed and inflation is set to zero. This stylized feature is coherent with price-stable policies of most developed countries and inflation rates near to the ZLB (1.2 % per year in average in the Eurozone over the period 2009-2019). Inflation would bring their set-up closer to a flexible price setting.

Demand for safe assets is then determined by the quantity of goods that Knightians received as dividends and as newborn endowment. Supply is determined by the value of Lucas trees after a negative productive shock and the securitization ability. In a flexible price setting, supply and demand equalize through interest rate or good price. But, assuming price rigidity and the existence of money, these price adjustments are constrained. It might be that, at full employment, the demand for safe assets is higher than supply for $r^{safe} = 0$. The Zero Lower Bound is binding and the only way for market to clear is a decrease of demand through an output below potential. The economy is in a Safety Trap.

The authors found that, in a safety trap, issuing safe public debt (backed on future capital taxation incomes) is a way to increase safe storage capacity of the economy and hence output. When we look at real world, this theoretical consideration is puzzling. Why is full-employment not restored in all countries with safe interest rates at the ZLB and significant risk premium? One of the above hypotheses, that we could relax, is the closed economy assumption. Do governments want to pledge future taxation revenues to increase world safe storage capacity rather than solely their national one?

This is the issue we want to tackle in this master thesis. We base ourselves on Caballero and Fahri (2018) and extend its model to an open economy with two symmetric countries: Home and Foreign. We want to think the first one as a developed country and the second as a developing one. We assume therefore Home to have an higher ability to issue safe assets and Foreign's Lucas trees to have higher expected returns. Both countries have the choice between financial integration and autarky. We will check under which conditions an Home government, willing to maximize domestic wealth, would choose financial integration, all else equal. Indeed, in our model, financial integration is not linked, for instance, to commercial openness or productivity as it may be in real world.

In section 2, after solving the model in closed economy, we look to the effects of financial integration in absence of public debt. First, we let both countries differ only regarding their securitization abilities: Home is supposed to produce more private safe assets in autarky than Foreign. Indeed, it is usually harder for financial institutions of emerging economies to issue safe assets. We assume the existence of a risk-premium and get the following results. Risk-averse agents lose from financial integration in Home because of a decrease in safe interest rate and/or in capacity utilization. In absence of safety trap, Neutrals in Home have to pay lower interest rate on their safe debt issuance and benefit from integration. The reverse is true in Foreign. But, when we include safety trap, foreign agents gain from an higher output after financial integration. In Home, output is not necessarily lower but, if the world ends up in a safety trap, it could prejudice Neutrals' wealth.

Then, we forget about redistributive effects of financial integration between Knightians and Neutrals. We focus on aggregate national wealth and add differences in expected return between foreign and domestic Lucas trees. Financial integration becomes a zero-sum games that can be beneficial either to Home or Foreign depending on parameter values. It is even a positive-sum game when, under autarky, domestic output is at potential and Foreign is in a safety trap.

In section 3, we assume both countries to have the same parameters and to be in a safety trap under autarky (as the world under financial integration). We introduce safe public debt as a way to increase safe storage capacity. By assumption, solely the domestic government can issue a quantity of debt denoted D. It has the choice to issue it under financial integration or autarky and to back it on future capital or labour incomes. Taxation is done without distortion costs.

If Home government is willing to maximize aggregate wealth of its nationals, it will choose either autarky (with taxation on capital or labour) or financial integration with capital taxation. As in Caballero and Farhi (2018), in absence of distortion costs, the level of debt issuance is likely to permit an exit from the safety trap. Moreover, we find that Home has no interest to increase the world safe storage capacity through a backing of its future labour incomes.

Thus, we assume Home to back its debt on capital revenues. We introduce distortion costs due to taxation and increasing more than proportionally in D. They are bearded by capital, affecting agents only through their assets holdings. Foreign and domestic investors detaining the same portfolios, distortion costs are bearded by both countries equally. Nevertheless, the first marginal units of public debt are issued with little distortions costs compared to the following ones. When the domestic government want its national safe saving capacity to increase of a given amount, it has to issue twice as much debt under financial integration than under autarky. The average distortion cost per unit of debt is then higher under financial integration. It follows that Home government would be better off under autarky.

At the end, if we suppose Lucas trees to have the same expected returns in both countries, financial integration is costly for Home in our set-up. It comes in conflict with real world observation, where developed countries are highly financially integrated and act as safe assets providers. Some gains from integration could explain this feature. In our analysis, we modelize one explanatory factor: risky investments may have higher expected returns in Foreign than in Home. We could think to other aspects as a close connection between financial and commercial integration or risk-sharing for example. We will outline some of these elements in our conclusion.

Related literature This master thesis inscribes itself in the literature on safe assets. This latter considers that assets, which are accepted at their face value "no question asked", should be studied apart from risky assets. Empirical studies have shown some specificities of safe assets as a relative stable share in total saving in the US (Gorton et al., 2012) or a convenience yield that cannot be explained by usual pricing theory (Krishnamurthy & Vissing-Jorgensen, 2012). To study safe assets in a more theoretical way, we use heterogeneity in risk aversion among agents as in Barro et al. (2014).

Our model will only focus on safe and risky assets. If money was in positive supply, it would be a perfect substitute for safe assets and therefore hold because of risk-aversion. The cash-in-advance constraint and the speculative motive, which are the two other drivers of liquidity preference, are indeed absent in our set-up. Nevertheless, we could have been interested to the relationships money/safe assets or money/risky assets.

For an analysis of the former, we refer to Gorton and He (2016). It investigates the trade-off, from a central banker point of view, to let more cash or more

Treasuries in the economy. At a given level of central bank liabilities, finding a satisfying equilibrium can be precarious. It can be that cash and Treasuries supplies should both increase: the former to avoid deflation and the second to prevent private creation of pseudo-safe Treasuries substitutes (as MBS).

An analysis of the relation money/risky assets can be found in the liquidity trap literature, from Keynes (1936)[1] and then the IS-LM model of Hicks (1937), defended in Krugman (2000), to more recent developments as Krugman (1998) or Christiano et al. (2011). This literature emphasizes traditionally the role of speculative considerations in liquidity preference.

We know from Tirole (1985), that, as long as $r = 0$, a bubble can be stable. In order to differentiate money from public debt, we assume the former to be always a bubble and the latter to be fully-backed by real revenues. There is hence no public bubble in our setting. We also rule out the possibility of private bubbles. They are less likely to be stable in the long-run since they suppose a large amount of investors to expect r remaining equal to zero forever.

We will assume in our study that the private sector is issuing the maximal quantity of safe assets it can. Gorton and Ordoñez (2014) argues that the private banking sector is tempted to issue more safe assets when demand is high. It leads banks to accept riskier collaterals, what increases financial instability. A similar mechanism is at work in Azzimonti and Yared (2019). The more private pseudo-safe assets are issued, the less sure they are. Our hypothesis of binding securitization constraint is therefore coherent with these studies.

Furthermore, we have been discussing liquidity and safety trap without specifying the duration of such episodes. Some economists think them as a new normal in a low-growth world and argue for secular stagnation. There has been a renewal of this literature after the 2008 financial crisis and especially since Summers (2014). Eggertsson and Mehrotra (2014) links secular stagnation to new characteristics of developed economies as an ageing population, an increase in inequality or a fall in the relative price of investment. Caballero et al. (2017) studies similar structural tendencies and associates them to a global safe asset shortage. Without entering the debate on secular stagnation, this literature allows us to consider long-lasting safety traps, what we do implicitly in our model.

Extending the investigation on safe assets to an open-economy framework raises an issue. How do we explain, that only a few number of countries are able to issue safe assets? Safe assets providers may be less risk averse than other countries (Farhi & Maggiori, 2018) or have better risk management technology (Gourinchas et al., 2010). For public debts, coordination problems may be one explanatory factor as in He et al. (2016). In presence of roll-over risk, an

[1]In his General Theory of Employment, Interest and Money, Keynes acknowledges that money could be hold for precautionary purposes. But he rules out the importance of this liquidity motive and writes: "liquidity-preferences due to the transactions-motive and the precautionary-motive are assumed to absorb a quantity of cash which is not very sensitive to changes in the rate of interest as such", in Chapter 13, II.

equilibrium is reached, when investors decide which debt(s) they consider as sure. A large amount of indebtedness may be an advantage in this setting, giving depth to a debt.

Safety could also be linked to the existence of international currencies. Their issuers might face a “new triffin dilemma”: either they issue enough safe assets, increasing their liabilities or they don’t and let their currency appreciate. Bordo and McCauley (2019) rejects the idea, that the USA would currently face this dilemma. On one side, US public debt is not the only way to issue safe assets in US dollars. On the other side, other currencies (and especially renminbi) have increased their aggregate supply of money since 2014 without backing it on dollars. We have to be cautious about these conclusions. The example of Switzerland, examined in Gourinchas and Rey (2016), shows us that a trade-off increasing exposure/currency appreciation may be at stake. It is even more interesting since Swiss safe assets are mostly private. We take it as an invitation to consider jointly public and private exposure as recommended in Reinhart and Rogoff (2015).

Moreover, in a situation of global safety or liquidity trap, there can be zero-sum games between countries to “export” their output gap. It is namely the case in Eggertsson et al. (2016). It describes secular stagnation as a contagious malady in open-economies frameworks. Caballero et al. (2016) introduces safe assets as a storage technology and money as a unit of account in a Mundell-Fleming model. It shows that a safe asset shortage situation can lead to a currency war. When a country is devaluating its currency more than others, it can export some of its “natural” output gap.

Lastly, this master thesis will lack of a fine analysis of the imbrications between public debt and production. Public debt can serve as liquidity provider for firms (Holmström & Tirole, 2001) or crowd out productive investments (Acharya & Dogra, 2020). Kahn (2019) argues empirically that over the recent period the first effect has dominated the second one. A finding that would be coherent with the safety trap analysis.

Our analysis involves two steps. First, Section 2 presents and solves the model without public debt. It assumes different parameter values in Home and Foreign. Then, section 3 forgets about these differences and introduces public debt. It solves the Home government maximisation problem by finding optimal debt issuance. Finally, section 4 concludes and outlines some benefits from debt and financial integration, which are missing in our set-up.

2 The Model without public debt

2.1 General description

Demographics Our setup is an OLG economy with continuous time and infinite horizon. Agents are born and die at hazard rate θ. Probability of dying is independent of agent's age. Population, a continuum of agents, which size is normalized to 1, is composed of two types of agents: Knightians, who are infinitely risk-averse and Neutrals, who are risk-neutral. Knightians represent a share α of the total population and Neutrals a share $1-\alpha$. We assume newborns and dying agents to be representative of the whole population.

Preferences Agents consume only when they die. We denote the death date of an agent σ_θ. As in Caballero and Farhi (2018), we define utilities of Knightians U_t^K and Neutrals U_t^N as:

$$U_t^K = \mathbb{1}_{\{t-dt<\sigma_\theta<t\}}c_t + \mathbb{1}_{\{t\leq\sigma_\theta\}}\min_t[U_{t+dt}^K]$$

and

$$U_t^N = \mathbb{1}_{\{t-dt<\sigma_\theta<t\}}c_t + \mathbb{1}_{\{t\leq\sigma_\theta\}}\mathbb{E}_t[U_{t+dt}^N]$$

Since one cannot be born and die exactly at the same time, storage technologies will be crucial with this preference setting. Moreover, θ can be understood as a propensity to consume out of wealth.

Aggregate Shock on output Aggregate risk comes from the possibility of a productive shock $\pi \in \{+;-\}$ on output that can be either negative (-) or positive (+). Potential output before the shock is equal to X. After a positive shock, it becomes μ^+X and, after a negative one, it becomes μ^-X, with $\mu^- < 1 < \mu^+$.

To know when the shock occurs and if it is positive or negative, we run two Poisson processes with probabilities λ^+ and λ^-. We write σ^+ and σ^- the two respective stopping times of these processes. Then, we define $\sigma = min\{\sigma^+;\sigma^-\}$, the time at which the shock occurs. If $\sigma = \sigma^+$, the shock is positive, otherwise it is negative.

Since we are dealing with Poisson processes, the probability that $\sigma^+ = \sigma^-$ is equal to zero. After the productive shock, there is no uncertainty any more. Knightians' and Neutrals' preferences become identical.

Endowment and Lucas trees Between t and $t+dt$, a quantity X_tdt of an homogenous good is distributed to agents. A share $(1-\delta)X_tdt$ is divided equally between all newborns of the time-span. The remaining δX_tdt are shared uniformly to a continuum of Lucas trees (normalized to 1) as dividends. δ can be interpreted as the share of capital and $1-\delta$ as the share of labor in the value added distribution.

Securitization process We assume that only Neutrals can hold Lucas trees. Neutrals can then issue and sell to Knightians state-contingent assets (with three different possible states: after a positive, a negative or no shock). An asset pays in every period a dividend, which solely depends on the state of the world. Securities do not protect against idiosyncratic death risk. Neutrals behave as banks: they collect savings from Knightians by selling assets, use collected resources to buy Lucas trees and redistribute wealth in further periods to Knightians since they have pledged a share of their future (state-contingent) cash-flows.

But this process is imperfect. A principal-agent problem (taken here as exogenous) forbids Neutrals to promise all their shock-dependent resources. In any state of the world, the maximal fraction of dividends that can be pledged is defined as ρ. We suppose $\rho > \alpha$, such that the securitization constraint is not binding in a world without aggregate risk.

Knightians being infinitely risk-averse, they value their portfolio at the lowest value it can take. If a portfolio has negative returns in one state of the world, a Knightian considers it as worthless. Indeed, it anticipates the case with negative interest rate and where its life-time would tend to infinity. Under this scenario, which has a probability tending to zero for $\theta > 0$, Knightian's wealth would indeed tend to zero.

To evaluate a portfolio, which has always non-negative interest rate, a Knightian anticipates its immediate death (no possibility to accumulate wealth in case of strictly positive dividends) in all possible future state of the world. At this death date, there are as many possible values of the portfolio as there are possible state of the world. A Knightian values the portfolio at the lowest of these possible portfolio prices.

Before a shock, there are three potential future states of the world. The three portfolio values in the three state of the world are perfect complement in the Knightians' utility function. The marginal benefit of one unit of wealth in a given state of the world is either one or zero.

Since Neutrals value positively every marginal increase of their wealth in a possible future state of the world, they sell to Knightians assets that have exactly the same value in every further periods and states. We can thus call them "safe assets". Furthermore, we assume the safe assets production to be constrained by Neutrals' securitization capacity after a negative shock.[2]

Money and Inflation Low inflation has been a political target in most developed countries since the 1980's. Voluntarist policies have actually succeeded to moderate price increases. We understand therefore low inflation more as a choice of economic framework[3] than as the result of technical barriers (as menu

[2]Formally, we state $\mu^-[1 + \frac{\lambda^+(\mu^+-1)+\lambda^-(\mu^--1)}{\delta\theta}] < 1$ and $\mu^-[1 + \frac{\alpha-\rho\mu^-}{\rho\mu^-}\frac{1-\delta}{\delta}] < 1$.

[3]For a further analysis of the French example, see Lemoine (2016), which emphasizes especially the role of public debt as a tool to achieve low-inflation.

costs for example). In order to stylize it in a simple way, we will assume the price of goods to be fixed to 1 (without loss of generality). It is an extreme assumption of price rigidity that could be relaxed in further studies.

In addition, we assume the existence of paper money[4]: a safe asset with a nominal rate of return $r = 0$. This asset will be in zero (net or gross) supply in our model. Nevertheless, its existence induces a Zero Lower Bound on nominal safe interest rates.

The combination of paper money and the absence of inflation forms an economic framework, where a 1:1 safe storage technology is guaranteed by the government. This point is crucial for our analysis. Without this double state intervention, there would be no Zero Lower Bound for real safe interest rate.

Capacity utilization Given price rigidities, it is possible that, before a productive shock, the goods market does not clear at potential output. To enable market clearing, we allow output to be below potential through $\xi \in [0; 1]$ the capacity utilization. Actual output is then equal to ξX. After a productive shock, output is always at potential, because of our hypothesis $\rho > \alpha$.

We assume capital, Knightians' and Neutrals' labour to be perfect complements. Concretely, it implies that when output is below potential, all types of revenues are below their potential values in the same proportions. Dividends are therefore equal to $\delta\xi X$, newborns goods endowments become $\alpha(1-\delta)\xi X$ for Knightians and $(1-\alpha)(1-\delta)\xi X$ for Neutrals.

Notice that the economy is never above potential ($\xi \leq 1$). Output is demand determined, $C_t = \xi_t X$, and a value of ξ below one can be interpreted as involuntary unemployment. There is no voluntary unemployment here since we didn't define any disutility of work.

Steady-states There are three steady-states in this model: a stochastic one before the productive shock and two possible deterministic steady-states after the shock π.

Under the stochastic steady-state, as long as no productive shock occurs, the composition and value of Knightians' and Neutrals' aggregate portfolios are fixed. When agents die, they exchange their portfolio against goods from newborns endowment and from living agents dividends. Composition of portfolios depends only on agents' type. The model aggregates cleanly since all living agents have the same death probability every period (no influence of age or type) and preferences are linear.

After a shock π, there is no aggregate risk any more and every asset is safe. This is unrealistic, but allows us to modelize a perception of risk backward. Differences in preferences between Knightians and Neutrals vanish. All newborns are receiving the same endowment and save at the same interest rate. Hence,

[4]Notice that we have taken a narrow defintion of paper money : one could think to paper money with $r < 0$ as the *Freigeld* of Silvio Gesell. Moreover, even without paper money, it would potentially be politically difficult for a government to not ensure a zero interest rate on bank deposits.

when the population share of agents born after the productive shock tends to 1, aggregate wealth of each agent's type converges to a share of total wealth, which depends only on demographic weights. Knightians' share converges to α and Neutrals' one to $1-\alpha$. Since total wealth takes different values after a negative or a positive productive shock, we have two possible deterministic steady-states.

2.2 Solving the model in closed economy

We define aggregate wealth of Knightians and Neutrals in the stochastic steady-state respectively W^K and W^N. For simplicity, we assume that Neutrals are always issuing the maximum of safe assets (i.e. in the limit of the securitization constraint). We then look if Neutrals are indeed selling all the issued safe assets to Knightians or if they hold some. We denote V^R and V^S the aggregate values of risky and safe assets before the shock. After a positive/negative shock, we write $V^R = V^{R+}$ or $V^R = V^{R-}$, even if risky assets have become safe. By definition, V^S remains constant in any state of the world and we have $V^S = V^{S-} = V^{S+}$. Finally, we write $W = W^K + W^N$, $V = V^R + V^N$ before a shock and $V^\pi = V^{R\pi} + V^{N\pi}$ thereafter.

Aggregate consumption is linked to aggregate wealth and death rate in the following way: $C_t = \theta W_t$. θ can be interpreted as a consumption over wealth ratio. Moreover, goods market clearing implies that consumption is equal to output in every period. We can therefore deduce a wealth-output relationship. Due to the asset market clearing condition $V = W$, we have in steady-state:

$$V = W = \frac{\xi X}{\theta} \qquad V^+ = W^+ = \mu^+ \frac{X}{\theta} \qquad V^- = W^- = \mu^- \frac{X}{\theta}$$

We have assumed the securitization constraint to be binding after a negative shock and get:

$$V^S = V^{S+} = V^{S-} = \rho\mu^- \frac{X}{\theta}$$

By definition $V = V^R + V^S$ in every state, hence:

$$V^R = (\xi - \rho\mu^-)\frac{X}{\theta} \qquad V^{R+} = (\mu^+ - \rho\mu^-)\frac{X}{\theta} \qquad V^{R-} = (\mu^- - \rho\mu^-)\frac{X}{\theta}$$

We define the interest rate on safe assets r_t and the expected one on risky assets r_t^{risky}. We assume the expected return on Lucas trees, taken as a whole, to be positive before a productive shock: $\delta\theta + \lambda^+(\mu^+ - 1) + \lambda^-(\mu^- - 1) > 0$.

Following Caballero and Farhi (2018), in the stochastic steady-state (and hence assuming no shock at t), we identify the subsequent wealth laws of motion:

$$\dot{W_t^K} = -\theta W_t^K + \alpha(1-\delta)\xi X + r_t W_t^K$$

$$\dot{W_t^N} = -\theta W_t^N + (1-\alpha)(1-\delta)\xi X + r_t(V^S - W_t^K) + r_t^{risky} V^R \\ - \lambda^+(V^{R+} - V^R) - \lambda^-(V^{R-} - V^R)$$

We find steady-state values by setting $\dot{W_t^K}$ and $\dot{W_t^N}$ both equal to zero.

Two regimes, four cases We have then two possibilities. Either the expected risky interest rate is strictly greater than the safe one or both are equal. In the first case, where $r_t^{risky} > r_t$, if a Neutral holds some safe assets, he would have interest to sell them and to buy risky assets. Consequently, in equilibrium, Neutrals detain only risky assets and Knightians hold all safe assets. We denote this situation as the constrained regime and Knightians are the marginal holders of safe assets ($W_t^K = V^S$).

In the second case, where $r_t = r_t^{risky}$, Neutrals are indifferent between risky and safe saving technologies. They are hence likely to hold some safe assets. We are in the unconstrained regime and Neutrals are the marginal holders of safe assets ($W_t^K < V^S$). Notice that it cannot be that $r_t > r_t^{risky}$. Otherwise, Neutrals would have interest to sell their risky assets to buy safe ones.

To identify in which situation we are, we look at the following safe asset shortage condition:

$$\frac{\alpha - \rho\mu^-}{\rho\mu^-}(1-\delta)\theta + \lambda^+(\mu^+ - 1) + \lambda^-(\mu^- - 1) \lesseqgtr 0$$

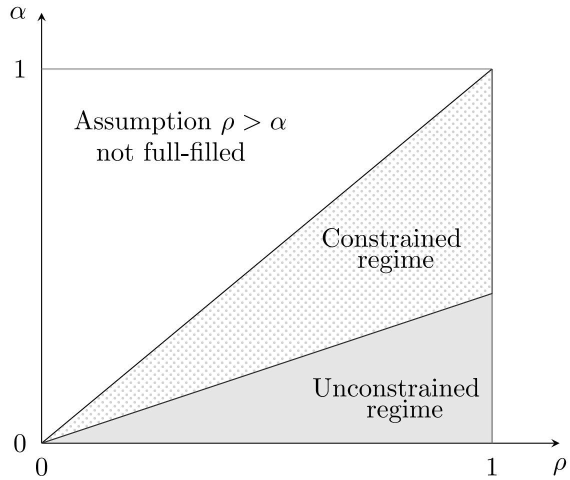
If the left-hand side is below zero, we are in the unconstrained case (i.e. without risk premium). Similarly, if it is above zero we are in the constrained case. The left-hand side is increasing in α and decreasing in ρ. The more Knightians there are, compared to Neutrals, or the lower the securitization capacity, the higher the chances to be in a constrained regime. We get this intuition graphically in Fig.1. Following the safe assets shortage condition, the line separating constrained and unconstrained regimes has a slope of $\mu^- \frac{(1-\delta)\theta - \lambda^+(\mu^+-1) - (\lambda^- - 1)}{(1-\delta)\theta}$.

In both regimes, we can define a "natural" real safe interest rate r^n for which output is at potential and safe asset market clears. It can be that r^n takes negative values. There is two ways to achieve negative returns in a flexible price setting: either negative nominal interest rates or increasing prices. In our setup, both are impossible and we would have $r = 0$ (ZLB is binding).

In the constrained regime, following Caballero and Farhi (2018), we have:

$$r^n_{constr} = \delta\theta - \frac{\alpha - \rho\mu^-}{\rho\mu^-}(1-\delta)\theta$$

Figure 1: Constrained and unconstrained regimes in the closed economy



We have seen in the presentation of our setting that, given Knightians' infinite risk-aversion, Neutrals want to issue "safe" assets, which have the same value whatever the state of the world. We have assumed the issuance of these assets to be limited by the securitization constraint after a negative productive shock. This is the reason why r^n_{constr} depends on μ^- (and not on μ^+ for example).

As all interest rates in our model, r^n_{constr} is increasing in δ, the capital share in wealth distribution, and in θ, the death rate (i.e. the propensity to consume out of wealth). Indeed, the more agents want to consume, the lower the demand for safe assets and the higher the interest rate. It reflects also safe storage scarcity. It is increasing in the securitization capacity ρ and decreasing in the demographic weight of Knightians α.

If r^n_{constr} is negative, it implies a demand for safe assets higher than supply when output is at potential and $r = 0$. Market can then only clear through quantity. The maximal capacity of the safe storage technology at $r = 0$ defines the maximum quantity of newborns' goods that can be exchange against safe assets. This latter must be equal to $\alpha(1-\delta)\xi X dt$ in a time-span dt and we derive ξ. Although, the storage capacities shortage being limited to safe assets, r^{risky} remains positive. This situation is defined as a safety trap.

In the unconstrained regime, safe and expected risky returns are equal. r^n takes negative values if, and only if, the expected return on Lucas tree is negative at $\xi = 1$. We assumed it to be always false. Nevertheless, if we relax our assumption[5], a negative r^n in unconstrained regime would correspond to a

[5]If the expected return on Lucas tree is lower than zero at $\xi = 1$, the safe asset shortage condition rewrites $\frac{\lambda^+\mu^+ + \lambda^-\mu^-}{\lambda^+ + \lambda^- - \delta\theta} - \frac{\rho\mu^-}{\alpha(1-\delta)} \lesseqgtr 0$. When the LHS is negative, the economy is constrained and in a liquidity trap. When it is negative, the economy is unconstrained and in

general shortage of assets, called liquidity trap.

To summarize, we have four subcases:

	Constrained regime ($W^K = V^S$)	Unconstrained regime ($W^K < V^S$)
$r^n < 0$	Safety trap	Liquidity trap
$r^n \geq 0$	Full-employment with risk premium	Full-employment without risk premium

The Safety trap A Safety trap occurs when the LHS of the safe asset shortage condition is greater than zero and that $r^n < 0$. Rewriting these two conditions as a restriction on ρ gives us:

$$\rho < min \left[\frac{\alpha(1-\delta)}{\mu^-} ; \frac{\alpha(1-\delta)\theta}{\mu^-[(1-\delta)\theta - \lambda^+(\mu^+ - 1) - \lambda^-(\mu^- - 1)]} \right]$$

Output is at potential if the totality of newborns' and dividend endowments can be exchanged against assets. In the constrained regime, there are two parallel asset market. The risky one, where risk-neutral newborns' endowments and Neutrals' dividends are exchanged against risky assets, sold by dying Neutrals. And the market for safe assets, where risk-averse newborns' endowments and Knightians' dividends are exchanged against safe assets, sold by dying Knightians.

If $r^n < 0$, when output is at potential, the market for safe assets clears if, and only if, dividends on safe assets are negative. In other word, Knightians' newborns would be able to exchange all their goods against safe assets, if safe assets supply would come from dying and living Knightians. The latter would give up a share of their wealth: a negative dividend.

We assumed that the existence of money rules out the possibility of negative dividends. Hence, if $r^n < 0$, dividends on safe assets would be equal to zero and not all risk-averse newborns' goods would be exchanged against assets. In our model, this can only be achieved through a ξ below 1.

To sum up, the securitization constraint and the total wealth level after a negative shock limit the quantity of safe assets that can be issued before a shock V^S. V^S defines then the supply side of the safe asset market. Since demand side is fully determined by Knightians newborns endowment at $r = 0$, this market clears through ξ following $\alpha(1-\delta)\xi X dt = \theta V^S dt$. Given our definition of capacity utilisation ξ, adjustment in quantities hit then all types of revenues. But its roots are to be find solely on the safe asset market.

---

a safety trap. The LHS is still increasing in α and decreasing in ρ.

Moreover, we have in unconstrained regime $r^n_{unconstr} = \delta\theta + \lambda^+(\mu^+ - 1) + \lambda^-(\mu^- - 1)$.

Our main issue to solve the equilibrium in a safety trap is then to find ξ. We know that $V^S = W^K$ since we are in a constrained regime. The quantity of safe assets that can be issued is constrained by the pledgeable wealth share after a negative shock $V^S = \rho\mu^- \frac{X}{\theta}$. Moreover, we know from its law of motion that Knightians' wealth, in the stochastic steady-state, is equal to $W^K = \frac{\alpha(1-\delta)\xi X}{\theta - r} = \frac{\alpha(1-\delta)\xi X}{\theta}$. We get hence:

$$W^K = \rho\mu^- \frac{X}{\theta} \qquad\qquad \xi = \frac{\rho\mu^-}{\alpha(1-\delta)} < 1$$

Since we are in the constrained regime, W^K is equal to the total value of safe assets. On the safe asset market, this wealth level ensures an assets supply $\theta W^K dt$ during a time span dt. In the meantime, on the demand side, the value of potential Knightians newborns' goods endowment is $\alpha(1-\delta)X dt$. In a safety-trap, $\alpha(1-\delta)X > \theta W^K$ and risk-averse newborns can exchange solely a share $\frac{\rho\mu^-}{\alpha(1-\delta)}$ of their goods against safe assets. This can be achieved, under our assumptions, only by setting $\xi = \frac{\rho\mu^-}{\alpha(1-\delta)}$. ξ depends therefore on the safe assets scarcity: it is increasing in ρ and decreasing in α.

Output (ξX) is equal to consumption, which is equal to wealth times the consumption over wealth ratio (θ). Consequently, a decrease in capacity utilisation is linked to a decrease in total wealth. Neutrals' aggregate wealth is thus equal to:

$$W^N = W - W^K = \frac{\xi X}{\theta} - \rho\mu^- \frac{X}{\theta}$$

We replace ξ by the value we have found above and get:

$$W^N = \left[\frac{1-\alpha(1-\delta)}{\alpha(1-\delta)}\right] \rho\mu^- \frac{X}{\theta}$$

Neutrals' aggregate wealth is increasing in ξ. It is penalized by safe asset scarcity through a reduced capacity utilization. In a safety trap, Neutrals' wealth is therefore decreasing in α and even increasing in ρ.

2.3 Financial integration

The objective of this subsection is to extend our previous model to an open economy with two countries, Home and Foreign, differing solely in their securitization capacities. We denote foreign parameters and variables with the superscript "*". For simplicity, we suppose both economies to be of equal size $X = X^*$. More generally, we assume all parameters to have the same value in both countries except ρ, which reflects the securitization process. We suppose $\rho^* < \rho$. Developing countries tend to issue less private safe assets, we can hence interpret Foreign as a developing country and Home as a developed one. Some explaining factors of a low ρ^* may be the volatility of currency value when inflation is high, less developed banking system or (partly self-fulfilling) low confidence of international investors.

We want to compare wealth between autarky and perfect financial integration. The latter is defined as the possibility for every domestic and foreign agent to buy and sell safe and risky assets of both countries without transaction costs or discrimination. We use the superscript ^{FI} to designate wealth after financial integration in one country. If it used for the total value of a kind of assets as with $V^{S,FI}$ or V^{FI}, it refers to an average: the entire world quantity of assets divided by two. In absence of superscript, we are studying autarky values.

Home is producing more safe assets than Foreign. This corresponds to an higher leverage of domestic Lucas trees and implies an higher volatility of Home's risky assets. We assume Neutrals of both countries to hold the same proportion of domestic and foreign risky assets. Since perfect financial integration implies no discrimination or transaction costs, this portfolio strategy is optimal. Absence of Home bias is refuted empirically, but is coherent with our benchmark definition of financial integration.

We focus our analysis on the stochastic steady-state. We will indeed never study possible transitions before the stochastic steady-state. Transitory effects could be due to higher initial endowments in one country or to the opening (or closing) of financial markets. Nevertheless, the government decisions we are looking at, as integration of financial markets or latter public debt issuance, are structural. We can therefore assume them to not be influenced by transitory effects. Moreover, we will not investigate the deterministic periods for now since, after a productivity shock, ρ has neither influence on steady-state wealth levels per type nor on aggregate national wealth.

We assume that there is only one productive shock π, which is of the same sign and occurs at $t = \sigma$ in Home and Foreign. There is hence no possibility of risk-sharing. Integration leads both economies to the same situation (since they have identical parameters except ρ): an isomorphism of the closed economy model with $\overline{\rho} = \frac{\rho+\rho^*}{2}$ instead of ρ. If both countries were not of same size, we would calculate a weighted mean for $\overline{\rho}$.

The isomorphism feature implies capacity utilization to be the same in Home and Foreign under financial integration. We denote ξ^{FI} this world capacity utilization. Foreign and domestic newborns' endowments are hence equal. We have assumed agents of the same type to have the same portfolio strategies independently of their nationality. Due to financial integration without cost or discrimination, the same portfolio structuration leads to the same return on capital in every period. One domestic agent and a foreign one, born at the same date and of the same type, would therefore have the same initial endowment and the same capital revenues thereafter (as long as both are still alive). Since $\theta = \theta^*$, we deduce $W^{K,FI} = W^{K,FI*}$ and $W^{N,FI} = W^{N,FI*}$.

Finally, given that the existence of a risk-premium is an historically widespread feature, we restrict our analysis to constrained regime cases.

Assumption 1: In autarky, in Home and Foreign, we have $\frac{\alpha-\rho\mu^-}{\rho\mu^-}(1-\delta)\theta + \lambda^+(\mu^+-1) + \lambda^-(\mu^- - 1) > 0$ for $\rho \in \{\rho; \rho^*\}$ and $\rho^* < \rho$.

Proposition 1: In the stochastic steady-state and under Assumption 1, Financial integration leads to a zero-sum game between risk-averse agents. Foreign Knightians increase their aggregate wealth of $\frac{\rho-\rho^*}{2}\mu^- \frac{X}{\theta}$ compared to autarky. Domestic Knightians' wealth decreases by the same amount.

Proof: Under Assumption 1, Knightians are the marginal holders of safe assets. $V^S = W^K$ in both countries in autarky and therefore also under financial integration. We have $\rho^* < \overline{\rho} < \rho$ and hence $V^{S*} < V^{S,FI} < V^S$. We deduce that $W^{K*} < W^{K,FI} < W^K$.

$$\text{Knightians' losses in Home:} \quad W^{S,FI} - W^S = \frac{\rho^* - \rho}{2}\mu^- \frac{X}{\theta} < 0$$

$$\text{Knightians' gains in Foreign:} \quad W^{S,FI} - W^S = \frac{\rho - \rho^*}{2}\mu^- \frac{X}{\theta} > 0$$

As long as we are in the constrained regime, Knightians' aggregate wealth is determined solely by Neutrals' safe assets issuance capacity. ρ being higher than ρ^*, Home is issuing more safe assets in autarky than Foreign. Financial integration leads to a level of safe assets issuance per country $V^{S,FI}$, which is the average between V^S and V^{S*}. In Foreign, there is hence more safe assets available per Knightian under Financial integration than under autarky. Foreign Knightians' wealth increases with integration. The reverse is true in Home.

Proposition 2: In the stochastic steady-state and under Assumption 1, Financial integration raises Domestic Neutrals' aggregate wealth of $\left[(\overline{\rho} - \rho^*)\mu^- + (\xi^{FI} - \xi)\right] \frac{X}{\theta}$ compared to autarky. Similarly, foreign Neutrals' aggregate wealth increases of $\left[(\overline{\rho} - \rho)\mu^- + (\xi^{FI} - \xi^*)\right] \frac{X}{\theta}$. Both effects can be either positive or negative depending on parameter values.

Proof: Under Assumption 1, Neutrals hold only risky assets and $V^R = W^N$ in both countries in autarky as under financial integration. We have to check our proposition in three scenarios. If there is no safety trap, a safety trap in both countries in autarky or a safety trap in autarky solely in Foreign. This last eventuality can then be divided into two subcases: either the world ends up in a safety trap or in full employment. Notice that, given $\rho^* < \rho$, it is impossible that a safety trap occurs in Home and not in Foreign.

Absence of safety trap Let us first look at the case where both economies are at their potential output. We have $V^{S*} < V^{S,FI} < V^S$ as shown previously and $V = \frac{X}{\theta}$. We deduce $V^R < V^{R,FI} < V^{R*}$ and hence $W^N < W^{N,FI} < W^{N*}$.

$$\text{Neutrals' gains in Home:} \quad W^{N,FI} - W^N = [(1-\overline{\rho}\mu^-) - (1-\rho\mu^-)]\frac{X}{\theta}$$
$$= \frac{\rho - \rho^*}{2}\mu^- \frac{X}{\theta} > 0$$

$$\text{Neutrals' losses in Foreign:} \quad W^{N,FI} - W^{N*} = \frac{\rho^* - \rho}{2}\mu^- \frac{X}{\theta} < 0$$

In absence of safety trap, $\xi = \xi^* = \xi^{FI} = 1$ and proposition 2 is full-filled. We have here a pure interest rate effect. As we have seen previously, Financial integration increases safe assets supply in Foreign and decreases it in Home. Since in both countries, safe assets demand remains unchanged, r increases in Foreign and decreases in Home compared to autarky. Neutrals don't save at r but financed themselves at this interest rate under assumption 1. We get hence a zero-sum game that benefit to Home Neutrals.

We have therefore two zero-sum games: one between Knightians that benefits to Foreign and one between Neutrals that benefits to Home. The political choice of an economic framework has here no consequences on aggregate domestic wealth, but on inequalities. A change in government preferences can shape differently market rules and finally wealth distribution.

If we assume, for example, that, in each country, a median voter takes the decision to open or not national financial markets. Integration is implemented if, and only if, both median voters benefit from it. It would require the median voter of Home to be a Neutral and the one of Foreign to be a Knightian.

Both countries in a safety trap We now look to the case, where both economies are below potential in autarky.

$$W^{N,FI} - W^N = \left[\frac{1-\alpha(1-\delta)}{\alpha(1-\delta)}\right]\frac{\rho^* - \rho}{2}\mu^- \frac{X}{\theta} < 0$$

Rearranging, we find our proposition 2:

$$\left[\frac{1-\alpha(1-\delta)}{\alpha(1-\delta)}\right]\frac{\rho^*-\rho}{2}\mu^-\frac{X}{\theta} = \frac{\rho-\rho^*}{2}\mu^-\frac{X}{\theta} + \left[\frac{1-\alpha(1-\delta)}{\alpha(1-\delta)}+1\right]\frac{\rho^*-\rho}{2}\mu^-\frac{X}{\theta}$$
$$= \left[(\overline{\rho}-\rho^*)\mu^- + (\xi^{FI}-\xi)\right]\frac{X}{\theta}$$

And similary,

$$W^{N,FI} - W^{N*} = \left[\frac{1-\alpha(1-\delta)}{\alpha(1-\delta)}\right]\frac{\rho-\rho^*}{2}\mu^-\frac{X}{\theta} > 0$$

In contrast to the previous case, we don't have any interest rate effect here. r is indeed always equal to zero in a safety trap. But we have an output effect. As we have seen in closed economy, in a safety trap, capacity utilization is increasing in safe assets supply. The more safe assets they are, the more Knightians' newborns goods can be exchanged against assets and the higher ξ.

Since $\rho > \rho^*$, there are more safe assets in Home than Foreign in autarky and $V^R < V^{R,FI} < V^{R*}$. It follows that $\xi < \xi^{FI} < \xi^*$. And since Neutrals' wealth is increasing in capacity utilisation, Foreign Neutrals benefit from integration at the expense of domestic ones. Financial integration leads consequently to two zero-sum games, one between Knightians and one between Neutrals, that both favor Foreign.

Foreign in a safety trap If Foreign is the only country in a safety trap in autarky, one more time, financial integration will increase safe assets supply in Foreign and reduce it in Home. This can lead to two situations. Either the increase in safe storage capacity in Foreign is sufficient to get Foreign out of the safety trap and world output is at potential. Or the domestic decrease in safe assets supply brings Home in a safety trap and world output is below potential. Safety trap can thus be contagious.

If the world ends up in a safety trap, foreign Neutrals' aggregate wealth is the same as in previous case. But effects on domestic Neutrals become ambiguous. On one side, they benefit from a lower r. This effect is lower than in the case without safety trap. Indeed, when the ZLB is binding, r cannot decrease any more. On the other side, domestic Neutrals suffer from a lower capacity utilization ξ compared to autarky. Formally, we have:

$$W^{N,FI} - W^{N} = \left[\frac{1-\alpha(1-\delta)}{\alpha(1-\delta)}\overline{\rho}\mu^- - (1-\rho\mu^-)\right]\frac{X}{\theta} \lesseqgtr 0$$

Since $\xi = 1$, proposition 2 is full-filled:

$$\begin{aligned}
&\left[\frac{1-\alpha(1-\delta)}{\alpha(1-\delta)}\overline{\rho}\mu^- - (1-\rho\mu^-)\right]\frac{X}{\theta}\\
=&\frac{\rho-\rho^*}{2}\mu^-\frac{X}{\theta} + \left[\frac{1-\alpha(1-\delta)}{\alpha(1-\delta)}\overline{\rho}\mu^- - (1-\rho\mu^- + \frac{\rho-\rho^*}{2}\mu^-)\right]\frac{X}{\theta}\\
=&\frac{\rho-\rho^*}{2}\mu^-\frac{X}{\theta} + \left[\frac{\overline{\rho}\mu^-}{\alpha(1-\delta)} - 1\right]\frac{X}{\theta}
\end{aligned}$$

In any manner, Neutrals' wealth increase will be lower than wealth decrease of domestic Knightians. Home government will not benefit from financial integration if it aims to maximize aggregate domestic wealth.

On the contrary, if world safe interest rate is strictly positive, domestic Neutrals benefit unambiguously from financial integration. Their aggregate wealth increases in the same way as in the scenario without any safety trap (through a decrease of their leverage costs). Effect on foreign Neutrals is now ambiguous. Capacity utilization raises, but they finance themselves at an higher price ($r^{K,FI} > r^K$):

$$W^{N,FI} - W^{N*} = \left[1 - \overline{\rho}\mu^- - \frac{1-\alpha(1-\delta)}{\alpha(1-\delta)}\rho^*\mu^-\right]\frac{X}{\theta} \lesseqgtr 0$$

Since $\xi^{FI} = 1$, our proposition 2 is also full-filled here and is therefore proved in its general statement. Furthermore, we notice that, when only one country is in a safety trap in autarky, Financial integration is a positive-sum game.

2.4 Adding difference in expected return

In the previous section, in Home, financial integration was mostly redistributive from Knightians to Neutrals and could only reduce total domestic wealth. Indeed, we didn't modelize any one of the traditional gains from financial integration.

To rectify this situation, we will now assume higher expected returns on risky investments in Foreign than in Home. Financial integration would then induce higher return for domestic Neutrals compared to autarky. This is coherent with our hypothesis that Foreign is a developing country and Home a developed one. For a descriptive analysis of the differences in expected returns between developed and developing countries, we refer to the "World investment report" of the UNCTAD (2019).

We modelize higher returns in Foreign by playing on probabilities of positive and negative shocks. Since our productive shock is highly stylized, it doesn't matter on which parameters we are playing to increase expected returns. We could also have changed $\mu^{+,*}$, $\mu^{-,*}$ or solely one $\lambda^{\pi,*}$.

We want to have $\lambda^{+} < \lambda^{+,*}$ and $\lambda^{-,*} < \lambda^{-}$. We let therefore productive shocks of both countries be of different signs. Nevertheless, we want productive shock to occur at the same time in Home and Foreign. Otherwise, we would get stochastic states of the world with large amount of safe assets.

Assumption 2: When $\sigma = \sigma^{-}$, we assume Foreign to be hit by a negative productivity shock with probability $1-\phi$ and by a positive one with probability ϕ, with $\phi \in [0;1]$.

This is equivalent to set $\lambda^{+,*} = \lambda^{+} + \lambda^{-}\phi$ and $\lambda^{-,*} = \lambda^{-}(1-\phi)$ without finding different σ for Home and Foreign.[6] Consequently, Foreign has an advantage in risky assets production (which have higher returns than domestic ones) and Home preserves its advantage in safe assets (since it can securitize an higher share of its Lucas trees). It introduces potential gains from integration for both countries. We assume the maximization problem of Home government to be :

$$\max_{\eta \in \{FI, aut\}} \mathbb{E}_0 \left[\int_0^{+\infty} \beta^t (W_t^{K,\eta} + W_t^{N,\eta}) dt \right]$$

with the economy being in stochastic steady-state at $t = 0$. We will not study the redistributive effects of financial integration in this section. Nevertheless, higher foreign expected return will reinforce Neutrals' benefits from financial integration in Home and Neutrals' losses in Foreign.

As long as both economies remain at the same productivity level, there is no difference in dividends between holding a foreign or a domestic risky asset. Assumption 2 can be beneficial to domestic agents only in the case where $\sigma = \sigma^{-}$ and a positive shock occurs in Foreign. At the time of a shock $\pi = -$ and $\pi^* = +$, a portfolio composed solely of risky domestic assets losses a fraction $\frac{V^R - V^{R-}}{V^R}$ of its value. In contrast, a portfolio composed solely of risky foreign assets multiply its value by a coefficient $1 + \frac{V^{R+*} - V^{R*}}{V^{R*}}$. At $t = \sigma$ and for a given level of leverage, domestic Neutrals are better off holding the half of foreign Lucas trees and the half of domestic ones rather than the totality of their national trees. They benefit from holding foreign assets and thus from financial integration.

After this productive shock, the foreign Lucas trees pay an aggregate dividend of $\delta\mu^{+}X$ each period, while domestic ones pay $\delta\mu^{-}X$. The aggregate value of foreign Lucas trees is therefore $\frac{\mu^{+}}{\mu^{-}}$ times higher than the domestic one. In

[6] Under this assumption, the open economy model is not an isomorphism of the closed economy where ρ, λ^{+} and λ^{-} would have been replace by $\frac{\rho+\rho^*}{2}$, $\frac{\lambda^{+}+\lambda^{+,*}}{2}$ and $\frac{\lambda^{+}+\lambda^{+,*}}{2}$. Indeed, there are some interactions between λ^{π} and ξ (which depends on ρ), as in the calculus of the expected risky rate of return, which invalidate the isomorphism conjecture.

the same time, newborns' aggregate endowment in Foreign, $(1-\delta)\mu^+X$, is $\frac{\mu^+}{\mu^-}$ times higher than the one in Home, $(1-\delta)\mu^-X$. It follows that, on the asset market, supply and demand are both $\frac{\mu^++\mu^-}{2\mu^-}$ times higher than in the case of financial integration with $\pi = -$ and $\pi^* = -$. Market clears hence at the same interest rate value, $\delta\theta$, and there is no gain compared to autarky for domestic agents born after the shock.

We can conclude that, in Home, a positive ϕ benefits solely to Neutrals living at $t = \sigma$. We will now try to quantify this benefit according to the government utility.

At $t = \sigma$, all agents of both countries are born before the shock. They had the same newborn endowment and the same return on assets depending on their type. We have hence:

$$W_\sigma^{FI} = W_\sigma^{FI*} = \frac{\mu^+ + \mu^-}{2} \times \frac{X}{\theta}$$

As t tends to infinity, the whole population tends to be born after the shock (for $\theta > 0$). Since domestic newborns receive only a share $\frac{\mu^-}{\mu^-+\mu^+}$ of total newborns' endowment after the shock, we have in steady-state (StSt):

$$W_{StSt}^{FI*} = \frac{\mu^+}{\mu^-} W_{StSt}^{FI} \qquad\qquad W_{StSt}^{FI} + W_{StSt}^{FI*} = \frac{(\mu^+ + \mu^-)X}{\theta}$$

And we can deduce W_{StSt}^{FI}:

$$W_{StSt}^{FI} = \frac{\mu^- X}{\theta}$$

In case of negative domestic shock and positive foreign one, W_{StSt}^{FI} is equal to W^- in autarky. We define $\widehat{W}_t^{FI}$ the wealth surplus induced by financial integration in a period $t > \sigma$, with $\widehat{W}_t^{FI} = W_t^{FI} - W_{StSt}^{FI}$.

It follows that at $t = \sigma$ and knowing $\pi = -$ and $\pi = +$, Home government values the total surplus as:

$$\int_\sigma^{+\infty} \beta^{t-\sigma} \widehat{W}_t^{FI} dt$$

Moreover, since W_{StSt}^{FI} do not depend on t, we have the following law of motion (with $\delta\theta$ the interest rate after a shock):

$$\dot{\widehat{W}}_t^{FI} = \dot{W}_t^{FI} = -\theta W_t^{FI} + (1-\delta)\mu^- X + \delta\theta W_t^{FI}$$

Rearranging, we get $\dot{\widehat{W}}_t^{FI} = -\theta(1-\delta)\widehat{W}_t^{FI}$. Knowing $\widehat{W}_\sigma^{FI}$ and hence $\dot{\widehat{W}}_\sigma^{FI}$, we derive a more general expression of $\widehat{W}_t^{FI}$ for each $t > \sigma$:

$$\widehat{W}_t^{FI} = \frac{\mu^+ - \mu^-}{2} \times \frac{X}{\theta} e^{-\theta(1-\delta)(t-\sigma)}$$

At $t = \sigma$, when the shock is negative in Home and positive in Foreign, Home government values the national wealth surplus due to financial integration as:

$$\frac{\mu^+ + \mu^-}{2} \times \frac{X}{\theta} \int_\sigma^{+\infty} \left[\beta e^{-\theta(1-\delta)}\right]^{t-\sigma} dt$$

This is decreasing in θ. Indeed, the lower the death rate, the more time is needed to replace the whole population by agents born after the productive shock. θ reflects the persistence of a wealth surplus above steady-state level compared to autarky. The latter being defined solely by newborns' endowment per capita and interest rate.

Conditional on the occurrence of a productive shock, the probability of $\pi = -$ and $\pi^* = +$ is equal to $\frac{\lambda^- \phi}{\lambda^+ + \lambda^-}$. This result, combined to the previous one, allow us to deduce the government optimal decision.

Proposition 3: Under assumptions 1 and 2, Home government would choose financial integration only if:

$$\gamma(\xi^{FI} - \xi)\frac{X}{\theta} - \gamma\lambda^-\phi\frac{\mu^+ - \mu^-}{2}\frac{X}{\theta}\frac{1}{Ln(\beta) - \theta(1-\delta)} \geq 0$$

with $\gamma \equiv \int_0^{+\infty}[\beta(1-\lambda^+ - \lambda^-)]^t dt = -\frac{1}{Ln[\beta(1-\lambda^+ - \lambda^-)]}$

The first term of this expression represents domestic losses due to a lower capacity utilization rate under financial integration than in autarky. It is always slightly negative since $\rho^* < \rho$. The second term corresponds to the transitory gains due to domestic investments in Foreign done before a shock $\pi = -$ and $\pi = +$. It is strictly positive.

Similarly, Foreign government with similar preferences would choose financial integration only if:

$$\underbrace{\gamma(\xi^{FI} - \xi^*)\frac{X}{\theta}}_{\substack{\text{gains from FI due to} \\ \text{higher output}}} + \underbrace{\gamma\lambda^-\phi\frac{\mu^+ - \mu^-}{2}\frac{X}{\theta}\frac{1}{Ln(\beta) - \theta(1-\delta)}}_{\substack{\text{losses from FI since higher} \\ \text{expected return on risky assets in Foreign}}} \geq 0$$

If countries are both in a safety trap or both at full capacity in autarky, financial integration is therefore a zero-sum game. It can be beneficial to both governments only if $(\xi^{FI} - \xi^*) > (\xi - \xi^{FI})$, that is, only if Home is at full capacity and Foreign in a safety trap under autarky. In this case, Home's output decreases less than Foreign's one increases. Integration becomes a positive-sum game. Notice that it is a necessary, but not sufficient, condition to ensure gains from integration to both countries.

3 Introducing public debt in a world safety trap

From the previous section, we know that a government with relatively high securitization capacity and low return on risky assets would always accept financial integration if the world ends up out of the safety trap. We want now to abstract from private securitization or expected risky return differences. We state $\rho = \rho^*$ and $\phi = 0$. We choose parameter values such that both countries end up in a safety trap in absence of further safe storage capacities. We introduce then public debt as a tool to counter this safe assets shortage. By assumption, solely the domestic government is able to issue safe public debt. It is coherent with the fact that almost only developed countries are issuing safe public debt.

Formally, Home government can once issue a stock (with aggregate value D) of infinitely-lived bonds. Wealth raised from issuing public debt is redistributed equally through (national) agents. Effects of this distribution vanish over time and have no influence on the stochastic steady-state. D remains constant in all periods and debt principal is never repaid. Indeed, we are not interested in debt as a way to trade inter-temporally, but as a tool to store value.

One unit of bond pays a coupon defined, in every period, by the safe interest rate. There is hence no difference between these bonds and private issued safe assets. It is as if the government was rolling-over its debt continuously (since interest rate adjusts constantly), but without roll-over risk since bonds are infinitely-lived. To pay interests on debt, taxes have to be raised. Taxation can be done either on capital or labour incomes. We will first study capital and then labour taxation without distortion. Which kind of debt issuance is optimal: under autarky or under financial integration? backed on capital or labour revenues? Then, we will focus on capital taxation and add distortion costs.

In order to compare the different frameworks in which debt interests are repaid, we have to use systematically the same preferences. We assume the maximization problem of Home government to be :

$$\max_{D,\eta\in\{FI,aut\}} \mathbb{E}_0 \left[\int_0^{+\infty} \beta^t \left(W_t^{K,\eta}(D) + W_t^{N,\eta}(D) \right) dt \right]$$

with the economy being in stochastic steady-state at $t = 0$. Under this assumption, Home doesn't take in account the transitory domestic wealth surplus due to bonds' issuance. Debt is hence valued only as a storage technology. Moreover, if there exists more than one D values solving the previous problem, Home government is assumed to choose the lowest one.

3.1 Capital taxation without distortion costs

To finance the payment of interest rates on public debt, Home administration can raise taxes on national Lucas trees. In this section, we assume a tax rate τ^K on dividends δX. We denote τ^K_+ and τ^K_- the tax rates after a positive or a negative shock. Since D is constant, there is no debt dynamics after a productive shock. τ^K varies under the influence of productive shock. Neutrals want to insure Knightians against the cost of these possible changes. Consequently, taxation will, in the first place, reduce dividends on risky assets.

We want to exclude the possibility for taxation to reduce private safe assets' dividends. After a negative shock, if the wealth share coming from former risky assets is null, we have a crowding-out of this type. In this situation, the dividend share dedicated to public debt can only grow at the expense of private safe assets' share. An increase in public debt would then be fully compensated by a decrease in private safe assets issuance. Thus, it would not raise the safe storage capacity as a whole.

This type of crowding-out is far from usual meaning of crowding-out, which implies a trade-off between real (and risky) investment and public debt. Here, it would require an unrealistic level of public debt to have crowding-out between safe assets. Indeed, private safe assets are not bubbles, but rather backed on sound businesses (the Lucas trees). We can therefore assume the absence of crowding-out. Formally, we state $\tau^K \leq 1 - \rho$ in every state of the world. This condition can only be binding after a negative shock and we rewrite it as $\tau^K_- \leq 1-\rho$. In the stochastic steady-state, Public debt hence decreases the value of risky private assets, but leaves the value of private safe assets unchanged.

As shown in Caballero and Farhi (2018), issuing debt with capital taxation and no distortion costs, is equivalent to increasing ρ. In a closed economy setting, they replace ρ by $v(\frac{D}{X}) = \rho + \frac{\theta}{\mu^-}\frac{D}{X}$. This holds as long as our no crowding-out assumption also holds. Public debt increases the securitization capacity and they get:

$$V^S = v(\frac{D}{X})\mu^- \frac{X}{\theta} = \rho\mu^- \frac{X}{\theta} + D$$

In an open economy, $\overline{\rho}$ is replaced by $v(\frac{D}{2X}) = \overline{\rho} + \frac{\theta}{\mu^-}\frac{D}{2X}$. V^S, the average safe assets value per country, becomes:

$$V^{S,FI} = v(\frac{D}{2X})\mu^- \frac{X}{\theta} = \overline{\rho}\mu^- \frac{X}{\theta} + \frac{D}{2}$$

We will now study the marginal effects of an increase in public debt on agents' wealth. In a closed economy setting, ρ has no impact on aggregate wealth after a productive shock. Moreover, domestic and foreign wealth levels are equals at $t = \sigma$ and remain equal thereafter since interest rates and labour incomes are the same in Home and Foreign. $\overline{\rho}$ has therefore no impact on aggregate domestic (or foreign) wealth after a productive shock. We will investigate the effects of an increase in public debt only at the stochastic steady-state. We will have to study them in a safety trap, but also out of the safety trap. Indeed, Home government might still have incentives to increase its debt, even after having reached full-employment.

World in a safety trap Since $V^{S,FI} = W^{K,FI}$, we get:

$$\frac{\partial W^{K,FI}}{\partial D} = \frac{\partial V^{S,FI}}{\partial D} = \frac{1}{2}$$

Then, we look at total wealth and risky assets:

$$V^{FI} = \frac{\xi X}{\theta} = \frac{v(\frac{D}{2X})\mu^-}{\alpha(1-\delta)}\frac{X}{\theta}$$

$$V^{R,FI} = V^{FI} - V^{S,FI} = \left(\frac{\overline{\rho}\mu^-}{\alpha(1-\delta)} - \overline{\rho}\mu^-\right)\frac{X}{\theta} - \left(1 - \frac{1}{\alpha(1-\delta)}\right)\frac{D}{2}$$

The effect of an increase in D on domestic Neutrals' wealth is hence:

$$\frac{\partial W^{N,FI}}{\partial D} = -\frac{1}{2}\left(1 - \frac{1}{\alpha(1-\delta)}\right) > 0$$

In a safety trap, every agent of both countries benefit from an increase in domestic public debt. Home government has hence interest to increase its public debt as long as the world is in a safety trap (in the limit of the crowding-out constraint). We can notice than the higher the share of Knightians α, the higher the positive effects on capacity utilization and therefore on Neutrals' wealth.

World outside of the safety trap We have to differentiate two cases here: either the world is in the constrained regime with full-employment or in the unconstrained regime.

Let us first look at the first situation. Effects of debt emission on $W^{K,FI}$ is the same as in a safety trap. On the contrary, Neutrals' wealth doesn't benefit anymore from a raise in ξ. We get:

$$V^{R,FI} = V^{FI} - V^{S,FI} = (1 - \rho\mu^-)\frac{X}{\theta} - \frac{D}{2}$$

and since $V^{R,FI} = W^{N,FI}$:

$$\frac{\partial W^{N,FI}}{\partial D} = -\frac{1}{2}$$

We have hence a zero-sum game and public debt leads to transfers from Neutrals to Knightians. Increasing the level of public debt has only redistributive effect here. In a Pareto efficiency perspective, Home government has no incentive to raise debt issuance. Nevertheless, we can perfectly think to a government that would use debt to transfer wealth from Neutrals to Knightians in order to decrease inequalities.

In the unconstrained regime, regardless on whether we are in a liquidity trap or at full-employment, an increase in public debt has neither influence on total wealth nor on its distribution. Indeed, on one side total wealth is distributed regarding demographic weights, which are exogenous. And, on the other side, we assumed output, which determines total wealth, to be at potential in the unconstrained regime.[7]

To conclude, if Home government is looking for the lowest debt value that maximizes total wealth of its nationals, it would choose a value D, such that domestic economy would precisely exit the safety trap (assuming the non-crowding-out assumption to not be binding). Under financial autarky, marginal effects of debt issuance would have been twice as large, but Home would have the same debt issuance policy (just exit the safety trap) and same aggregate wealth. We get the following optimal debt issuances:

$$D^{K,aut} = \left[\alpha(1-\delta) - \rho\mu^-\right]\frac{X}{\theta} \qquad D^{K,FI} = \left[\alpha(1-\delta) - \overline{\rho}\mu^-\right]\frac{2X}{\theta}$$

3.2 Labour taxation without distortion costs

In the previous section, we have assumed that taxes, necessary to pay interests, were bearded by capital. With this taxation strategy, debt issuance led to new safe storage possibilities and a decrease in domestic private risky assets value. It had no effect on labour incomes. The state was "securitizing" domestic capital and swapped risky assets for safe ones. But what would happen if labour revenues would be taxed to reimburse debt? We want to address this issue now. After a productive shock, a tax rate τ^L_π is raised on labour incomes, that is on

[7] Even if we don't do this no-liquidity trap assumption, ξ doesn't depend on the securitization capacity of the economy and we have $\xi^{FI} = max\left\{\frac{\lambda^+\mu^+ + \lambda^-\mu^-}{\lambda^+ + \lambda^- - \delta\theta}\ ;\ 1\right\}$

newborns endowment. This second option reduces future labour incomes in order to increase future capital revenues, allowing issuance of safe assets. Indeed, an asset today is a claim on financial flows tomorrow.

Under autarky, after a productive shock π, wealth is equal to $\frac{\mu^\pi X}{X}$. It is given by output and D has hence no impact on aggregate wealth in a deterministic steady-state. In the stochastic steady-state, domestic public debt has, in a safety trap, the same effects as under capital taxation (indeed $r = 0$ implies no taxation). D is therefore chosen to just exit the safety trap. In this way, every marginal unit of public debt raises both Neutrals' and Knightians' wealth. We have:

$$D^{L,aut} = \left[\alpha(1-\delta) - \rho\mu^-\right] \frac{X}{\theta}$$

After a shock π, output $\mu^\pi X$ is distributed as follows: $\delta\mu^\pi X$ as private dividends, $(1-\delta)(1-\tau^L_\pi)\mu^\pi X$ to newborns and $\tau^L_\pi(1-\delta)\mu^\pi X$ as interests on public debt. Total wealth being equal to $\frac{\mu^\pi X}{\theta}$, we get the following equations:

$$r^{L,aut}_\pi(D) = \theta\delta + \theta(1-\delta)\tau^L_\pi(D)$$

$$\tau^{L,aut}_\pi(D) = \frac{r^{L,aut}_\pi(D)}{(1-\delta)\mu^\pi X} D$$

We solve and get:

$$\tau^{L,aut}_\pi(D) = \frac{\theta\delta D}{(1-\delta)(\mu^\pi X - \theta D)}$$

$$r^{L,aut}_\pi(D) = \frac{\theta\delta\mu^\pi X}{\mu^\pi X - \theta D}$$

We can derive an endogenous restriction on debt emission since we need $(1-\delta)\mu^\pi X \geq r^{L,aut}_\pi(D) \times D$ for each π: $D^{L,aut}_{max} = \frac{(1-\delta)\mu^- X}{\theta}$. We assume this constraint to not be binding.

After a productive shock, interest rate is increasing in D. Higher public debt level redistributes wealth from the short-lived agents to the long-lived ones, but not between type of agents. This effect is of little interest for us since our demographic process is unrealistic. Nevertheless, we can notice that public debt backed on labour revenues increases the capital share in wealth distribution through an increase in interest rate.

We now compare autarky to the situation under Financial integration. Similarly, we calculate domestic taxation and interest rate after a productive shock and get:

$$\tau_{\pi}^{L,FI}(D) = \frac{\theta\delta D}{(1-\delta)(\mu^{\pi}X - \theta\frac{D}{2})}$$

$$r_{\pi}^{L,FI}(D) = \frac{\theta\delta\mu^{\pi}X}{\mu^{\pi}X - \theta\frac{D}{2}}$$

Interest and taxation rates are, once again, increasing in D, but at a lower rate. We have now: $D_{max}^{L,FI} = \frac{2(1-\delta)\mu^{-}X}{\theta(1+\delta)}$

All agents born before the productive shock have received the same initial endowment whatever their country. Moreover, under financial integration and given our assumptions on portfolios, they have the same returns on their savings (conditional on their type). Since $\alpha = \alpha^*$ and $\theta = \theta^*$, it follows that, at $t = \sigma$, that is, when all agents are born before the shock, domestic and foreign national wealth are equal. Their value is $\frac{\mu^{\pi}X}{\theta}$ in both countries as in the closed economy.

After the shock, all agents save at the interest rate $r_{\pi}^{L,FI}(D)$. But, for $D > 0$ domestic newborns are taxed on their initial wealth and receive a lower endowment than in Foreign. It follows that within the population born after the shock, foreign agents' wealth will be more important than domestic one. As t tends to infinity, all agents are born after the shock and we are in a steady-state where $W^{L,FI} < W^{L,FI*}$.

We want to check this formally. For $t > \sigma$:

$$\dot{W}_t(D) = -\theta W_t + (1-\delta)(1-\tau_{\pi}^{L,FI}(D))\mu^{\pi}X + r_{\pi}^{L,FI}(D)W_t$$

Given $D_{max}^{L,FI}$, we know that $\mu^{\pi}X - \frac{\theta D}{2} - \theta\delta\mu^{\pi}X$ is positive and therefore that $\dot{W}_t(D)$ is decreasing in $W_t(D)$. There is hence only one steady-state value for W and W_t converges toward it after a productive shock. This steady-state value is the following:

$$W_{StSt,\pi}^{L,FI}(D) = \frac{(1-\delta)(\mu^{\pi}X - \theta\frac{D}{2}) - \theta\delta D}{\theta[(1-\delta)\mu^{\pi}X - \theta\frac{D}{2}]}\mu^{\pi}X$$

We deduce from this expression that $W_{StSt,\pi}^{L,FI}(0) = \frac{\mu^{\pi}X}{\theta}$ and $\frac{\partial W_{StSt,\pi}^{L,FI}(D)}{\partial D}$ is of the sign of $-[\frac{1}{2} + \frac{\theta\frac{D}{2}}{(1-\delta)\mu^{\pi}X - \frac{\theta D}{2}}] < 0$. The higher the public debt, the higher the taxes on newborns and the lower domestic wealth in the deterministic steady-states. A positive D level leads to positive net exports from Home to Foreign in every period after a shock.

With labour taxation, under financial integration, domestic aggregate wealth is equal to $\frac{\mu^{\pi}X}{\theta}$ only at $t = \sigma$ or for every t in the trivial case where $D = 0$. Otherwise, for every periods $t > \sigma$, wealth is below its value under autarky with labour taxation.

A positive D means a decrease in newborns endowment and an increase in assets supply. In autarky, the decrease in labour income was fully compensated by a raise of capital revenues. Under Financial integration, this compensation is imperfect. Indeed, the costs of debt issuance (a reduction in newborns endowment) are fully bearded by domestic agents, while the benefits (more assets and therefore an higher interest rate) are shared between both countries.

Capital versus Labour taxation Based on previous analysis, we can now compare two options to back debt: taxation on capital or labour revenues.

The first option leads to a "securitization" of the economy: domestic risky assets value diminish and, on the other side, safe assets are created. Since we have assumed public debt issuance to be done at zero distortion/transaction costs, this swap in assets' type has no effect on wealth level after a productive shock and increases output in the stochastic steady-state.

Under financial integration, the debt level that is needed to exit the safety trap is twice as high as in autarky. It is not a problem without distortion costs and as long as our safe non-crowding-out constraint is not binding. It leads domestic Neutrals to seek for risky investments in Foreign and foreign Knightians to invest in domestic safe assets. An intuition, which is close to the one of exorbitant privilege and exorbitant duty of safe assets providers (see Gourinchas et al., 2010). Costs and benefits from debt issuance are shared equally between countries since debt influences solely portfolio returns.

This is not the case with labour taxation, which impacts also newborn endowment. Under autarky, after a productive shock, this taxation choice has no impact on aggregate wealth. Nevertheless, it increases interest rate and decreases newborns' endowment. It influences therefore the value added distribution, not between Knightians and Neutrals, but between short- and long-lived agents.

Under financial integration, backing future labour incomes decreases domestic aggregate wealth after a productive shock. For any r, world assets supply increases compared to the case without debt, because of a raise in future capital revenues. This shift in the supply curve benefits to both countries. On the demand side, for any r, domestic newborns are demanding less assets due to the decrease of their endowment. Foreign demand remains stable since foreign newborns aren't taxed. The supply effect dominates in Foreign and the demand effect dominates in Home; It leads to a decrease in domestic aggregate wealth compared to autarky. Cost and benefits from debt issuance are hence not shared under labour taxation.

To conclude, under financial integration, it seems preferable for a government to back its debt on future national capital revenues rather than on labour incomes. That is the reason why we will now study only public debt backed on capital revenues.

3.3 Public debt with distortive capital taxation

Until now, Home government could back its future capital revenues without friction. This was unrealistic and we want to introduce distortion costs due to taxation. We assume them to be bearded by Lucas trees owners through a reduction in dividends.

Assumption 4 After a productive shock $\pi \in \{-;+\}$, distortionary effects of taxation reduce domestic dividends of $\frac{\eta\theta}{\mu^\pi X}D^2$, with η a distortion factor.[8]

We suppose thus distortion costs to be increasing more than proportionally in public debt. It is a more realistic feature than a proportional increase and will lead to more interesting results (no corner solution).

Under assumption 4, world output becomes $2\mu^\pi X - \frac{\eta\theta}{\mu^\pi X}D^2$ after a productive shock π. Since distortion costs are assumed to be fully bearded by a reduction in dividends, r is decreasing in D. To calculate it, we divide dividends by total assets value and get:

$$r^\pi(D) = \frac{2\delta\mu^\pi X - \frac{\eta\theta}{\mu^\pi X}D^2}{\frac{2\mu^\pi X - \frac{\eta\theta}{\mu^\pi X}D^2}{\theta}} = \frac{2\delta\theta\mu^\pi X - \frac{\eta\theta^2}{\mu^\pi X}D^2}{2\mu^\pi X - \frac{\eta\theta}{\mu^\pi X}D^2}$$

$r^\pi(D)$ is decreasing in debt over output. Indeed, the more debt per unit of output, the more taxation per Lucas tree and the more distortion. $r^\pi(D)$ is therefore decreasing in D and increasing in μ^π or X. To avoid safe interest payment and distortion costs to be higher than potential dividends, we assume η small enough such that $\left(\rho\frac{\mu^- X}{\theta} + D\right) r^\pi(D) + \frac{\eta\theta}{\mu^\pi X}D^2 < \delta\mu^\pi X \ \ \forall\pi \in \{-,+\}$.

We derive the government problem of 3.1 adding distortion costs. Maximization problem of domestic government remains, with the economy in the stochastic steady-state at $t = 0$:

$$\max_{D,\eta\in\{FI,aut\}} \mathbb{E}\left[\int_0^{+\infty} \beta^t W_t^\eta(D)dt\right]$$

We could also think to an Home government willing to maximize total world wealth with:

[8] We could have adopted a taxation distortion of the form $\tau^{K,cost} = \tau^{K,r} + \frac{\tau^{K,r2}}{2}$, with $\tau^{K,costs}$ the sum of the potential dividend share that goes to debt interests payment and the one that is lost due to distortions and $\tau^{K,r}$ the share of potential dividends received by the government to pay interest.

Nevertheless, it would have led to the same intuitions with more complicated calculus, reducing the readability of final results.

$$\max_{D,\eta \in \{FI,aut\}} \mathbb{E}\left[\int_0^{+\infty} \beta^t \left(W_t^{\eta}(D) + W_t^{\eta *}(D)\right) dt\right]$$

Interestingly, both preferences lead to the same maximization problem under financial integration. As in section 3.1, agents are affected by taxation and distortion costs only through their portfolios. Since agents are assumed to hold the same assets in Home and Foreign, costs of debt are shared equally between countries. The maximization problem of Home government under financial integration rewrites:

$$\max_{D} \; \gamma W^{FI}(D) + (1-\gamma)\left[\frac{\lambda^+}{\lambda^+ + \lambda^-} W^{FI,+}(D) + \frac{\lambda^-}{\lambda^+ + \lambda^-} W^{FI,-}(D)\right]$$

with

$$\gamma \equiv \int_0^{+\infty} [\beta(1-\lambda^+ - \lambda^-)]^t dt = -\frac{1}{Ln[\beta(1-\lambda^+ - \lambda^-)]}$$

$$W^{FI}(D) = \frac{\xi^{FI}(\frac{D}{2})X}{\theta}$$

$$W^{FI,+}(D) = \frac{\mu^+ X - \frac{\eta\theta}{2\mu^+ X} D^2}{\theta} \qquad W^{FI,-}(D) = \frac{\mu^- X - \frac{\eta\theta}{2\mu^- X} D^2}{\theta}$$

To solve this maximization problem, we first assume the optimal quantity of public debt to be too low to get the world out of the safety trap. Under this assumption, we have:

$$D = \frac{\gamma}{1-\gamma} \times \frac{\lambda^+ + \lambda^-}{\lambda^+\mu^- + \lambda^-\mu^+} \times \frac{\mu^+\mu^- X}{2\eta\alpha(1-\delta)}$$

Since it is possible that a lower quantity of debt would be enough to exit the safety trap, we rewrite:

$$D^{FI} = \min\left\{\left[\alpha(1-\delta) - \overline{\rho}\mu^-\right]\frac{2X}{\theta} \; ; \; \frac{\gamma}{1-\gamma} \times \frac{\lambda^+ + \lambda^-}{\lambda^+\mu^- + \lambda^-\mu^+} \times \frac{\mu^+\mu^- X}{2\eta\alpha(1-\delta)}\right\}$$

We investigate now the same problem with Home in autarky. Domestic government solves:

$$\max_{D} \; \gamma W^{aut}(D) + (1-\gamma)\left[\frac{\lambda^+}{\lambda^+ + \lambda^-} W^{aut,+}(D) + \frac{\lambda^-}{\lambda^+ + \lambda^-} W^{aut,-}(D)\right]$$

with

$$W^{aut}(D) = \frac{\xi^{aut}(D)X}{\theta}$$

$$W^{aut,+}(D) = \frac{\mu^+ X - \frac{\eta\theta}{\mu^+ X} D^2}{\theta} \qquad W^{FI,-}(D) = \frac{\mu^- X - \frac{\eta\theta}{\mu^- X} D^2}{\theta}$$

and we get, similarly as under financial integration,

$$D^{aut} = \min\left\{ \left[\alpha(1-\delta) - \overline{\rho}\mu^-\right] \frac{X}{\theta} \ ; \ \frac{\gamma}{1-\gamma} \times \frac{\lambda^+ + \lambda^-}{\lambda^+\mu^- + \lambda^-\mu^+} \times \frac{\mu^+\mu^- X}{2\eta\alpha(1-\delta)} \right\}$$

In any case, D is optimal when marginal cost of debt equalizes marginal benefit. Moreover, total gains and costs of debt are both shared equally between countries under financial integration. We have therefore $D^{aut} = D^{FI}$ as long as Home and world stay in a safety trap after debt issuance. It follows that domestic capacity utilization in autarky is higher (or equal) to the one under financial integration and Home government is better off under autarky.

When distortion costs are increasing more than proportionally in public debt, being able to share these costs with other debt holder countries is not enough to reproduce capacity utilization of the closed economy. The first units of bonds are issued with relatively small distortion effects. They offer a low-cost way to increase the safe storage capacity of an economy. Financial integration leads to the sharing of this advantage with Foreign.

Remark : if both countries could issue debt, they would both issue

$$D = \min\left\{ \left[\alpha(1-\delta) - \overline{\rho}\mu^-\right] \frac{X}{\theta} \ ; \ \frac{\gamma}{1-\gamma} \times \frac{\lambda^+ + \lambda^-}{\lambda^+\mu^- + \lambda^-\mu^+} \times \frac{\mu^+\mu^- X}{2\eta\alpha(1-\delta)} \right\}$$

This would correspond to the optimal debt emission. Safe storage capacity per Knightian would be the same as in closed economy. There would be no free-riding or strategic interaction. Indeed, under capital taxation, benefits and costs of debt emissions are shared perfectly through portfolios.

When would Home choose financial integration? Providing safe saving capacities to Foreign is always costly for Home. Nevertheless, domestic government could have an interest to financial integration if this cost is compensated by the gains we have studied in section 2. We take a simple example to illustrate the continuity between our analyses with and without public debt.

If we assume, under autarky, Home output to be at potential in constrained regime and Foreign to be in a safety trap, Home benefits from financial integration only if:

$$\underbrace{\gamma(\xi^{FI}(D^{FI})-1)\frac{X}{\theta}}_{\text{losses from FI due to lower output}} - \overbrace{\gamma\lambda^{-}\phi\frac{\mu^{+}-\mu^{-}}{2}\frac{X}{\theta}\frac{1}{Ln(\beta)-\theta(1-\delta)}}^{\text{gains from FI since higher expected return on risky assets in Foreign}}$$

$$-\underbrace{(1-\gamma){D^{FI}}^{2}\left(\frac{\lambda^{+}}{\lambda^{+}+\lambda^{-}}\frac{\eta}{\mu^{+}X}+\frac{\lambda^{-}}{\lambda^{+}+\lambda^{-}}\frac{\eta}{\mu^{-}X}\right)}_{\text{losses from distortion due to taxation}} \geq 0$$

4 Conclusion

A world safety trap is a situation of global safe assets shortage. Under these circumstances, financial integration comes with costs for Home in our model.

We have first proved that, when $\rho > \rho^{*}$, there was a cost linked to the sharing of domestic private safe saving capacities. Then, we have abstracted from differences in private securitization and have concentrated ourselves to the following question. Is there a cost, for a domestic government, to increase world safe storage capacity through backed debt issuance? In absence of distorting taxation, we have seen that domestic government do it at zero cost if it pledges its future capital incomes (rather than its labour revenues). Nevertheless, once we have introduced distortion costs increasing in D, financial integration has been costly for Home.

Be financially integrated and share safe storage capacities is hence likely to have a cost for developed countries in a safety trap. Once this conclusion has been drawn, we have to examine the gains from financial integration or debt issuance that can compensate this cost. Indeed, how do we explain that the current world safe asset providers remain financially integrated and issue (safe) public debt hold around the world?[9]

We can first think to traditional gains from financial integration. Home could benefit from risk-sharing or from attractive investment opportunities in Foreign. We have incorporated this last feature in our set-up. Under certain parameter values, Home became then a net beneficiary from integration. Moreover, there may be a close link between financial and commercial integration. Developed countries would then accept to bear some costs of financial integration against gains from trade. Similarly, financial exchanges could impact output level, if they are associated to technology transfers.

[9]Notice that in real world, there exists a continuum of possible configurations between financial autarky and full integration as we have defined them. The domestic temptation of financial autarky described above could translate in a will to issue safe assets reserved to nationals (passbook savings accounts as the the Livret A in France or pay-as-you-go retirement system for instance).

One other response element could be an historical one. In its current form, financial integration is a product of the 1980's, a time where almost nobody was anticipating binding ZLB and even less safety trap. As shown in Lemoine (2016), foreign holding of public debt was thought as a disciplining tool. This integration wave has lead to an high level of imbrication between foreign and domestic balance sheets. It may then be complicated to return to autarky without important disorganization costs.

Based on our analysis of the redistributive effects of financial integration, an additional factor could be that governments are not trying to increase aggregate national wealth. As shown in section 2.3, a Sovereign protecting Neutrals' interests only may be willing to integrate Home, even at the expense of total domestic wealth. The cost of integration would be bearded by Knightians. Knowing the connivance between political and business powers, this cannot be totally ruled out.

Finally, our analysis has considered public debt solely as a backed safe storage capacity. But it is also an inter-temporal trade. The pursuit of immediate revenues at the expense of future ones is a usual driver of indebtedness, especially in the Public Choice literature. Depending on government present bias, this motive could explain a sizeable share of public debt issuance.

More interestingly, it could be that public debts are not fully backed and are partially bubbles. These latter being more or less stable depending on trust in the Sovereign's institutions or in the issuance currency. Thinking to public debt as an hybrid object between money (a perfect bubble) and a fully backed assets seems us to be an interesting start point for further researches.

References

Acharya, S., & Dogra, K. (2020). *The side effects of safe asset creation* (tech. rep.). Federal Reserve Bank of New York.

Azzimonti, M., & Yared, P. (2019). The optimal public and private provision of safe assets. *Journal of Monetary Economics*, *102*, 126–144.

Barro, R. J., Fernández-Villaverde, J., Levintal, O., & Mollerus, A. (2014). *Safe assets* (tech. rep.). National Bureau of Economic Research.

Bordo, M. D., & McCauley, R. N. (2019). Triffin: Dilemma or myth? *IMF Economic Review*, *67*(4), 824–851.

Caballero, R. J., & Farhi, E. (2018). The safety trap. *The Review of Economic Studies*, *85*(1), 223–274.

Caballero, R. J., Farhi, E., & Gourinchas, P.-O. (2016). Safe asset scarcity and aggregate demand. *American Economic Review*, *106*(5), 513–18.

Caballero, R. J., Farhi, E., & Gourinchas, P.-O. (2017). Rents, technical change, and risk premia accounting for secular trends in interest rates, returns on capital, earning yields, and factor shares. *American Economic Review*, *107*(5), 614–20.

Christiano, L., Eichenbaum, M., & Rebelo, S. (2011). When is the government spending multiplier large? *Journal of Political Economy*, *119*(1), 78–121.

Eggertsson, G. B., & Mehrotra, N. R. (2014). *A model of secular stagnation* (tech. rep.). National Bureau of Economic Research.

Eggertsson, G. B., Mehrotra, N. R., Singh, S. R., & Summers, L. H. (2016). A contagious malady? open economy dimensions of secular stagnation. *IMF Economic Review*, *64*(4), 581–634.

Farhi, E., & Maggiori, M. (2018). A model of the international monetary system. *The Quarterly Journal of Economics*, *133*(1), 295–355.

Geis, A., Kapp, D., Kristiansen, K., et al. (2018). Measuring and interpreting the cost of equity in the euro area. *Economic Bulletin Articles*, *4*.

Gorton, G., & He, P. (2016). *Optimal monetary policy in a collateralized economy* (tech. rep.). National Bureau of Economic Research.

Gorton, G., Lewellen, S., & Metrick, A. (2012). The safe-asset share. *American Economic Review*, *102*(3), 101–06.

Gorton, G., & Ordoñez, G. (2014). Collateral crises. *American Economic Review*, *104*(2), 343–78.

Gourinchas, P.-O., & Rey, H. (2016). *Real interest rates, imbalances and the curse of regional safe asset providers at the zero lower bound* (tech. rep.). National Bureau of Economic Research.

Gourinchas, P.-O., Rey, H., & Govillot, N. (2010). *Exorbitant privilege and exorbitant duty* (tech. rep.). Institute for Monetary and Economic Studies, Bank of Japan.

He, Z., Krishnamurthy, A., & Milbradt, K. (2016). What makes us government bonds safe assets? *American Economic Review*, *106*(5), 519–23.

Hicks, J. R. (1937). Mr. keynes and the" classics"; a suggested interpretation. *Econometrica: journal of the Econometric Society*, 147–159.

Holmström, B., & Tirole, J. (2001). Lapm: A liquidity-based asset pricing model. *the Journal of Finance*, *56*(5), 1837–1867.

Kahn, R. J. (2019). Corporate demand for safe assets and government crowding-in. *SSRN 3286717*.

Keynes, J. M. (1936). *The general theory of interest, employment and money*. MacMillan.

Krishnamurthy, A., & Vissing-Jorgensen, A. (2012). The aggregate demand for treasury debt. *Journal of Political Economy*, *120*(2), 233–267.

Krugman, P. (1998). It's baaack: Japan's slump and the return of the liquidity trap. *Brookings Papers on Economic Activity*, *1998*(2), 137–205.

Krugman, P. (2000). How complicated does the model have to be? *Oxford Review of Economic Policy*, *16*(4), 33–42.

Lemoine, B. (2016). *L'ordre de la dette: Enquête sur les infortunes de l'etat et la prospérité du marché*. La découverte.

Reinhart, C. M., & Rogoff, K. S. (2015). Financial and sovereign debt crises: Some lessons learned and those forgotten. *Journal of Banking and Financial Economics*, (2 (4)), 5–17.

Summers, L. H. (2014). Us economic prospects: Secular stagnation, hysteresis, and the zero lower bound. *Business economics*, *49*(2), 65–73.

Tirole, J. (1985). Asset bubbles and overlapping generations. *Econometrica: Journal of the Econometric Society*, 1499–1528.

UNCTAD. (2019). World investment report of the united nations conference on trade and development.